A study of optimal dual frames for erasures

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Abstract

Frames have demonstrated their significant utility in data transmission by virtue of their redundant features, which facilitate the reconstruction of data with minimal errors even in the presence of erasures and distortions. These applications have naturally prompted inquiries into the identification of optimal dual frames or dual pairs that can provide superior approximations to the original signals. Broadly, there are two categories of investigations related to optimal dual frames:

1. The first category involves to characterize optimal dual frame of a given frame.

2. The second category focuses on the existence and characterization of a dual pair that minimizes, among all dual pairs, the maximum error operator's measure obtained by considering the various possible locations of a fixed number of erasures.

In our work, we meticulously characterize the spectrally optimal dual frame for two erasures. We identify optimal dual pairs using diverse error measures, including the Frobenius norm, spectral radius, and numerical radius. Our investigation extends to probabilistic erasure models, exploring their behavior under operator norm, spectral radius, and their averages. We delve into the realm of probabilistic optimal dual frames and dual pairs within this framework. Our investigations reveal that equiangular tight frames and their canonical duals often exhibit optimality among dual pairs. Consequently, we explore the conditions under which equiangular tight frames exist in Hilbert spaces, contributing to a comprehensive understanding of their existence in both real and complex Hilbert spaces.

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