ON SOLVING NONLINEAR ILL-POSED PROBLEMS WITH APPLICATION TO PARAMETER IDENTIFICATION PROBLEMS

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We consider nonlinear inverse problems of the type $F(x) = y$, where F is a continuous nonlinear operator between Hilbert spaces X and Y. The problems are ill-posed in the sense that the solution does not depend continuously on the data. In many practical applications, what we have will be a noisy data y^δ:$\|y^\delta - y\| \leq \delta$; $\delta > 0$ and hence, such problems require regularization techniques so that the solution x can be estimated. Some of the commonly implemented regularization techniques are Landweber method, steepest descent method, Levenberg-Marquardt method, Gauss-Newton method, etc. However, the convergence analysis of these schemes require restrictive assumptions on the Fréchet derivative which are difficult to satisfy. Hence, we explore simplified version of these iterative schemes where the Fréchet derivative needs to be calculated only once at the initial point. We also prove their convergence and convergence rate with weaker assumptions. In addition, we also explore parameter identification problems associated with partial differential equations which is an important category of nonlinear ill-posed problems and hence, can be treated in a special manner. We propose three modified iterative schemes for the same, namely, steepest descent, Levenberg-Marquardt and Gauss-Newton method, where we use an approximation of the Fréchet derivative which is much simpler in terms of calculation and analysis. We also implement all the proposed schemes numerically and compare it with the standard methods as well. In addition, we also explore an electrical impedance tomography problem and estimate the conductivity distribution using some of the proposed iterative schemes.

References

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