Exponent analogues of Schur's theorem (with) Part II: Existence of non-inner automorphisms of order p in finite nonabelian p-groups

Komma Patali

Let G be a group. A well-known theorem of Schur states that if the central quotient $G/Z(G)$ of G is finite, then the commutator subgroup $\gamma_2(G)$ is finite. The Schur multiplier of G is the second homology group $H_2(G, \mathbb{Z})$ of G with integer coefficients. A long-standing conjecture of Schur states that $\exp(H_2(G, \mathbb{Z})) \mid \exp(G)$ for every finite group G. In Part I of this talk, we investigate a problem analogous to the theorem of Schur. We prove that $\exp(\gamma_2(G)) \mid \exp(G/Z(G))$ for p-groups of class less than or equal to $p+1$. We prove Schur's conjecture for certain classes of finite p-groups, which includes finite p-groups of class at most p, odd order p-groups of class at most 5, and finite metabelian p-groups of class at most $2p-1$. We provide bounds on the exponent of the Schur multiplier. Let G be a finite p-group and S be a Sylow p-subgroup of $\mathrm{Aut}(G)$ with $\exp(S) = q$. We bound $\exp(G)$ by a function of q.

In 1973, Berkovich proposed that every finite nonabelian p-group admits a non-inner automorphism of order p. In Part II of this talk, we prove this conjecture for a subclass of finite p-groups with a cyclic center and finite p-groups of coclass 4 and 5 ($p \geq 5$).