Operator Means, Wasserstein mean and logarithmic mean of positive definite matrices and unipotent matrices

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Abstract

Kubo and Ando in 1980s have introduced operator mean of positive operators to generalize the notion of mean for positive numbers. Since their pioneer work on two-variable operator means, many different construction schemes of multi-variable geometric means have been developed. In this talk, we discuses two kinds of the least squares mean, Cartan mean and Wasserstein mean, on the cone of positive definite Hermitian matrices. We define the Wasserstein mean of $n \times n$ unipotent matrices by solving its corresponding matrix equation, and study its fundamental properties. Under a binomial expansion for the weighted geometric mean of unipotent matrices, we provide the explicit formula of two-variable Wasserstein means. Furthermore, we show that the multi-variable Wasserstein mean and Log-Euclidean mean are different, by finding their explicit forms for $n = 3$. We define the logarithmic mean of unipotent matrices using the integral representation of the logarithmic mean and the common limit of skewed iteration of the arithmetic and geometric means. Lastly, we show that both logarithmic means are equal.