Frame wavelets of local fields with no dual frame wavelets

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Abstract

A field K equipped with a topology is called a local field if both its additive and multiplicative groups are locally compact abelian.

A sequence $\{x_k\}_{k=0}^{\infty}$ in a separable Hilbert space H is said to be a frame of H if there exist constants A and B satisfying $0 < A \leq B < \infty$ such that

$$A\|x\|^2 \leq \sum_{k=0}^{\infty} |\langle x, x_k\rangle|^2 \leq B\|x\|^2 \quad \text{for all } x \in H.$$

A function $\psi \in L^2(K)$ is called a frame wavelet if $\{\psi_{j,k} : j \in \mathbb{Z}, k \geq 0\}$ is a frame of $L^2(K)$, where

$$\psi_{j,k}(x) = q^{\frac{j}{2}} \psi\big(\mathfrak{p}^{-j}x - u(k)\big), \quad j \in \mathbb{Z}, k \geq 0.$$

Here $\mathfrak{p}$ is a prime element of K, q is an appropriate integer, and $\{u(k) : k \geq 0\}$ is a complete set of distinct coset representatives of the unit ball in K.

In this talk, we will construct an example of a frame wavelet in a local field of positive characteristic which has no dual frame wavelet. Further, we will show that the corresponding space of negative dilates is shift-invariant.