ℓ-Regular Partitions/Multipartitions through the Lens of Theta Functions and Hecke Eigenforms

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A partition of a positive integer n is a non-increasing sequence of positive integers whose sum is n. The members of the sequence are called parts. For an integer $\ell \geq 2$, a partition of n is said to be ℓ-regular if none of its parts is divisible by ℓ.

Let $n = a_1 + a_2 + \cdots + a_r$ be a partition of n. Suppose, in turn, $\lambda^{(i)}$ is a partition of a_i for each $i \in \{1, 2, \ldots, r\}$. We call the ordered tuple $(\lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(r)})$ as a r-multipartition of n. If, for each $i \in \{1, 2, \cdots, r\}$, $\lambda^{(i)}$ is a ℓ-regular partition, then $(\lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(r)})$ is said to be a ℓ-regular r-multipartition of n.

We use the notations $b_\ell(n)$ and $B_\ell^{(r)}(n)$ to denote the number of ℓ-regular partitions and ℓ-regular r-multipartitions of n, respectively. Our talk will revolve around some congruences and divisibility properties satisfied by $b_\ell(n)$ and $B_\ell^{(r)}(n)$ for different values of ℓ and r. We will mainly focus on the following 3 topics:

1. infinite families of congruences satisfied by $b_\ell(n)$ modulo ℓ for $\ell \in \{17, 23\}$ and $b_{65}(n)$ modulo 13.
2. exact criteria on n for 3-divisibility of $b_9(n)$ and $b_{27}(n)$.
3. infinite families of congruences satisfied by $B_\ell^{(r)}(n)$ for different values of ℓ and r.

We will mostly concentrate on the tools and techniques used while proving the results rather than diving into explicit proofs.