A STUDY ON GAUGE THEORY AND PARALLEL TRANSPORT IN PRINCIPAL 2-BUNDLES OVER LIE GROUPOIDS.

ADITTYA CHAUDHURI

Abstract. For the last few decades or so, higher gauge theories provided frameworks for describing the dynamics of string-like extended "higher dimensional objects." Typically, they involve an appropriately categorified version of a smooth fiber bundle equipped with a connection structure that induces a notion of parallel transport consistent with this categorification. The precise description of the categorified objects depends mainly on the framework's purpose.

In my doctoral work, we introduce a notion of a principal 2-bundle over a Lie groupoid. For such principal 2-bundles, we have produced a short exact sequence of VB-groupoids, namely, the Atiyah sequence. Two notions of connection structures viz. strict connections and semi-strict connections on a principal 2-bundle arising, respectively, from a retraction of the Atiyah sequence and a retraction up to a natural isomorphism, have been introduced and have been described in terms of Lie 2-algebra valued 1-forms on the total Lie groupoids. An existence criterion for the connections on a principal 2-bundle over a proper, étale Lie groupoid is proposed. We studied the action of the 2-group of gauge transformations on the category of strict and semi-strict connections. We also observed an extended symmetry of the category of semi-strict connections.

We characterized certain subclasses of these principal 2-bundles by proving a Lie 2-group torsor version of the well-known one-one correspondence (due to Grothendieck) between fibered categories and pseudofunctors. This correspondence also enabled us to extend a particular class of principal 2-bundles to be defined over differentiable stacks. We show that the differential geometric connection structures combine nicely with an underlying fibration structure of a principal 2-bundle over a Lie groupoid. This interrelation allows us to produce a notion of parallel transport in the setup of principal 2-bundles over Lie groupoids along a particular class of Haefliger paths. We construct the corresponding parallel transport functor that jointly generalizes the classical notion of the thin fundamental groupoid of a manifold and the parallel transport functor of a principal bundle in the traditional setup. We also show that our theory of parallel transport in a 2-bundle framework will induce a similar one in the setup of VB-groupoids.

2010 *Mathematics Subject Classification.* Primary 53C08, Secondary 22A22, 58H05.

Key words and phrases. principal 2-bundles, Atiyah sequence, connections, gauge transformations, Lie groupoids.