Ph. D. Thesis Defence

ON GAUGE THEORY AND PARALLEL TRANSPORT IN PRINCIPAL 2-BUNDLES OVER LIE GROUPOIDS

By

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Abstract

My doctoral study investigates an interplay between some ideas in traditional gauge theory and the fibered category theory. We accomplish the same by introducing a notion of a principal Lie 2-group bundle over a Lie groupoid, and studying its connection structures, gauge transformations, and parallel transport. We investigate the underlying fibered category structure of our principal Lie 2-group bundle. This results in a statement and the proof of a Lie 2-group torsor version of the well-known one-one correspondence (due to Grothendieck) between fibered categories and pseudofunctors. As its corollary, we extend a subclass of our principal 2-bundles to be defined over the differentiable stack represented by the base Lie groupoid. We construct a short exact sequence of VB-groupoids, namely, the Atiyah sequence associated to our principal 2-bundles. As a splitting of the Atiyah sequence, and a splitting up to a natural isomorphism, we obtain two notions of connection structures, viz. strict connections and semi-strict connections, respectively. We describe these connections as Lie 2-algebra valued 1-forms on the total Lie groupoids. The underlying fibration structure of the 2-bundle provides an existence criterion for strict and semi-strict connections. We study the action of the 2-group of gauge transformations on the groupoid of strict and semi-strict connections, and interestingly, we observe an extended symmetry of semi-strict connections. Our study culminates into a theory of parallel transport on our principal 2-bundles. The most exciting ingredient of this theory is the notion of parallel transport along 'thin homotopy classes of Haefliger paths'. This notion reflects a decent interplay between fibrations and gauge theoretic connection structures. It shows that the theory of Lie 2-group bundles and their connection structures produces a well-defined diffelogically smooth parallel transport functor, defined on a 'thin fundamental groupoid of a Lie groupoid'. Finally, we apply our results to produce a notion of parallel transport along Haefliger paths in the framework of VB-groupoids. The thesis work is primarily based on [1] and [2].

References:

1. Saikat Chatterjee and Adittya Chaudhuri. Parallel transport on a Lie 2-group bundle over a Lie groupoid along Haefliger paths. arXiv preprint arXiv:2309.05355, 2023.
2. Saikat Chatterjee, Adittya Chaudhuri, and Praphulla Koushik. Atiyah sequence and gauge transformations of a principal 2-bundle over a Lie groupoid. Journal of Geometry and Physics, 176:Paper No. 104509, 29, 2022.

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