The size of the Schur multiplier of special p-groups and other classes of p-groups using graph theory

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Abstract

Let G be a group. The Schur multiplier of G is the second homology group $H_2(G, \mathbb{Z})$ of G with integer coefficients. Mathematicians have devoted considerable effort over the years to determining bounds on the order of the Schur multiplier for various classes of groups. In particular, bounds have been obtained for the size of the Schur multiplier of special p-groups of rank 2 and $\binom{d}{2}$. In this talk, we obtain bounds for the size of the Schur multiplier of p-groups of arbitrary rank k, where $2 \leq k \leq \binom{d}{2}$, using methods from graph theory. This general bound gives the bounds obtained for 2 and $\binom{d}{2}$. For this, we generalize a result by Igor Rivin. The author proved that in a graph with $\binom{r}{2}$ edges, the maximum number of triangles is at most $\binom{r}{3}$. We generalize this result to graphs with a fixed number of edges $n \in \mathbb{N}$. We will also investigate improvements on the bound when additional structure is imposed on the group. We also improve bounds for groups of nilpotency class 3. Using this bound, we improve the bound on the size of the Schur multiplier of groups of coclass $r = n - c$ where p^n is the order of the group and c is the nilpotency class of G.