

Exponent analogues of Schur's theorem (with)
Part II: Existence of non-inner automorphisms of
order p in finite p -groups

This thesis consists of two parts:

- (I) Exponent analogues of Schur's theorem.
- (II) Existence of non-inner automorphisms of order p in finite p -groups.

By a well-known theorem of Schur, if the central quotient of a group is finite, then the commutator subgroup is finite. The Schur multiplier of a group G is the second homology group $H_2(G, \mathbb{Z})$ of G with integer coefficients. The first part of the thesis is concerned with a problem analogous to the above theorem of Schur. We prove that $\exp(\gamma_2(G)) \mid \exp(G/Z(G))$ for certain classes of groups. Schur's exponent conjecture states that for every finite group, the exponent of Schur multiplier divides the exponent of the group. We establish Schur's exponent conjecture for certain classes of finite p -groups, and we provide bounds on the exponent of the Schur multiplier. Let G be a finite p -group and S be a Sylow p -subgroup of $\text{Aut}(G)$ with $\exp(S) = q$. We bound $\exp(G)$ by a function of q . In 1973, Berkovich proposed that every finite nonabelian p -group admits a non-inner automorphism of order p . The second part of the thesis is concerned with finite nonabelian p -groups for which this conjecture holds.

References

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