TOEPLITZ OPERATORS ON THE DIRICHLET SPACE AND HIGHER ORDER WEIGHTED DIRICHLET SPACES

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ABSTRACT. The aim of the talk is to study a class of Toeplitz operators on the Dirichlet space and higher order weighted Dirichlet spaces. In the first part, we focus on Toeplitz operators induced by bounded harmonic functions φ on the open unit disc \mathbb{D} , for which the measure

$$\left|\frac{\partial\varphi}{\partial z}\right|^2dA(z)$$

is a Carleson measure for the Dirichlet space. We establish several algebraic properties of these operators, obtaining results analogous to those of Brown and Halmos in their classical work "Algebraic properties of Toeplitz operators". In particular, we show that the operator identity

$$T_{\overline{z}}TT_z = T$$

characterizes all Toeplitz operators on the Dirichlet space of the aforementioned symbol class.

In the second part of the talk, we briefly introduce higher order weighted Dirichlet spaces and discuss preliminary results concerning the behaviour of multiplication-by-z operators. This part of the work is ongoing, and we conclude with some directions for further investigation.

Date: September 2025.