MIXED LOCAL NONLOCAL OPERATORS AND SINGULARITY: A MULTIPLE-SOLUTION PERSPECTIVE

We investigate the existence of multiple positive solutions for the following Dirichlet boundary value problem:

$$
\begin{aligned}
-\Delta_p u + (-\Delta_p)^s u &= \lambda \frac{f(u)}{u^\beta} \text{ in } \Omega \\
u &> 0 \text{ in } \Omega \\
u &= 0 \text{ in } \mathbb{R}^N \setminus \Omega
\end{aligned}
$$

where Ω is an arbitrary bounded domain in $\mathbb{R}^N$ with smooth boundary, $0 \leq \beta < 1$ and $f : [0, \infty) \to (0, \infty)$ is a non-decreasing C^1-function which is p-sublinear at infinity and satisfies $f(0) > 0$. By employing the method of sub- and supersolutions, we establish the existence of a positive solution for every $\lambda > 0$ and that of two positive solutions for a certain range of the parameter λ. In the non-singular case, we further apply Amann's fixed point theorem to show that the problem admits at least three positive solutions within this range of λ. The mixed local-nonlocal nature of the operator and the nonlinearity pose challenges in constructing sub- and supersolutions, however, these are effectively addressed through the operator's homogeneity.