ON REGULARITY RESULTS FOR NONHOMOGENEOUS ELLIPTIC PROBLEMS

Ritabrata Jana

This talk focuses on regularity theory for nonlinear elliptic problems involving nonlocal and mixed local-nonlocal operators.

In the first part of the talk, we study the weighted boundary regularity for weak solutions to the fractional (p, q)-Laplacian equation with zero Dirichlet condition:

$$(-\Delta)_p^s u + (-\Delta)_q^s u = f(x) \quad \text{in } \Omega, \text{ and } u = 0 \text{ in } \Omega^c,$$

where Ω is a $C^{1,1}$ bounded domain, $0 < s < 1$, and $2 \leq p \leq q < \infty$. For nonnegative data, we show that $u/d_\Omega^s \in C^\alpha(\overline{\Omega})$ for some $\alpha \in (0, 1)$, using a nonlocal boundary Harnack method and a novel barrier construction. The result extends to sign-changing bounded data for suitable range of s.

In the second part, we examine the Hölder regularity of solutions to equations involving a mixed local-nonlocal nonlinear nonhomogeneous operator $-\Delta_p + (-\Delta)_s^q$ with singular data, under the minimal assumption that $p > sq$. The regularity result is twofold: we establish interior gradient Hölder regularity for locally bounded data and boundary regularity for singular data. We prove both boundary Hölder and boundary gradient Hölder regularity depending on the degree of singularity.