Convergence of adaptive FEM for OCP governed by fourth-order PDE

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This talk focuses on the convergence of adaptive finite element methods (AFEM) for a class of optimal control problems (OCP) governed by the fourth-order partial differential equations (PDE). For the finite element approximation of OCP governed by the biharmonic equation with L^2 sources, we employ the nonconforming Morley finite element to discretize the state and adjoint variables. For the control variable, we consider both the variational and piecewise-constant discretization. Subsequently, we discuss three variants of OCP involving Dirac measures governed by the biharmonic equation: OCP with point sources, point-wise tracking OCP, and point-wise tracking OCP with point sources. Due to the lack of continuity of the Morley element beyond the nodes, we incorporate an enriching operator into the right-hand side of the discrete state equation to ensure the well-posedness of the discrete problem. In addition, we study the distributed OCP governed by a fourth-order PDE. Here, we employ the conforming hierarchical Argyris finite element for the discretization of the state and adjoint variables, and both the variational and piecewise constant discretization for the control variable. For all the above problems, we establish error equivalence results that relate the total error in the OCP to the errors in suitably chosen auxiliary PDE. These equivalence results are the foundation for deriving both a priori and a posteriori error estimates. The adaptive algorithm: *solve*, *estimate*, *mark*, and *refine* generates a sequence of discrete solutions using this a posteriori error estimator. Its optimal rate of convergence is ensured by verifying the standard adaptivity axioms: *stability* (A1), *reduction* (A2), *discrete reliability* (A3), and *quasi-orthogonality* (A4). The axioms (A1), (A2), and (A4) guarantee the R-linear convergence of the a posteriori error estimator, while (A3) establishes the optimality of Dörfler's marking strategy. Together, these results confirm the quasi-optimality of AFEM for the considered OCP. We also present numerical experiments to validate the theoretical results and provide additional insights into the problems.