Ph. D. Thesis Defence

Location of Eigenvalues of Matrix Polynomials and Rational Matrices

By

Mr. Shrinath Hadimani

Abstract

Polynomial root finding is a classical problem with applications in many branches of mathematics. A natural generalization of this problem is to consider polynomials with complex matrix coefficients, known as matrix polynomials, where one is interested in eigenvalues of such polynomials. These eigenvalues come out as roots of the determinant of matrix polynomials. The problem of finding eigenvalues and eigenvectors for matrix polynomials is referred to as polynomial eigenvalue problem, abbreviated as PEP. Matrix polynomials can also be viewed as matrices with scalar polynomial entries. Similarly, matrices with entries that are scalar rational functions are known as rational matrices, and the corresponding problem of finding their eigenvalues and eigenvectors is called the rational eigenvalue problem, abbreviated as REP. Determining the exact eigenvalues of matrix polynomials and rational matrices is challenging, hence they are often approximated using iterative methods. Consequently, establishing bounds on the eigenvalues is crucial for making an initial guess, which influences the convergence rate of the iteration.

In this talk, we determine the location of eigenvalues of complex and quaternion matrix polynomials, as well as complex rational matrices. We begin by proving that complex matrix polynomials with doubly stochastic or Schur stable matrix coefficients have eigenvalues confined within specific annular regions. We then discuss the notion of (hyper)stability of complex matrix polynomials and extend the same to right quaternion matrix polynomials in order to obtain location of their right eigenvalues relative to certain subsets of the set of quaternions. In particular, we prove that right eigenvalues of a right quaternion matrix polynomial lie between two concentric balls of specific radii in the set of quaternions centered at the origin. A generalization of the Eneström-Kakeya theorem to quaternion matrix polynomials is obtained as an application. We also deduce hyperstability of certain univariate quaternion matrix polynomials via stability of certain multivariate quaternion matrix polynomials.

Furthermore, we derive bounds on the moduli of eigenvalues of arbitrary and special type of complex rational matrices. For a given arbitrary complex rational matrix $R(\lambda)$, we associate a block matrix C_R, whose blocks consist of the coefficient matrices of $R(\lambda)$, as well as a scalar real rational function $q(x)$, whose coefficients are the norms of the coefficient matrices of $R(\lambda)$. We prove that a zero of $q(x)$ which is greater than the moduli of all the poles of $R(\lambda)$ is an upper bound on the moduli of eigenvalues of $R(\lambda)$. Moreover, by using a block matrix associated with $q(x)$, we establish bounds on the zeros of $q(x)$, which in turn yield bounds on the moduli of eigenvalues of $R(\lambda)$. For eigenvalues of a special type of complex rational matrix $T(\lambda)$, we obtain an upper bound by applying the Bauer-Fike theorem on the associated block matrix C_T. We derive a lower bound in terms of a zero of a scalar real rational function $p(x)$, associated with $T(\lambda)$, using Rouché's theorem for matrix-valued functions. Other upper bounds, by applying numerical radius inequality to a block matrix $C_{\mathbb{S}}$, associated with a scalar real rational function $q(x)$ corresponding to $T(\lambda)$ are also obtained.

References

[1] Basavaraju, P., Hadimani, S. and Jayaraman, S., "Bounds on the moduli of eigenvalues of rational matrices", *Results Math.*, 79(5) (2024), Paper No. 206.

[2] Basavaraju, P., Hadimani, S. and Jayaraman, S., "Stability of quaternion matrix polynomials", Linear Algebra, Matrices and their Applications, *Contemporary Mathematics*, AMS (Edited by Surender Kumar Jain, Manjunatha Prasad Karantha, Steve Kirkland, Vinay Madhusudanan, Srinivasa Siva Rama Krishna Rao Taduri), accepted for publication, *arXiv:2407.16603*.

[3] Basavaraju, P., Hadimani, S. and Jayaraman, S., "Spectral bounds for certain special type of rational matrices", under review, *arXiv:2302.02894*.

Date: April 16, 2025

Time: 4.00 PM, Venue: PSB 1104