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A QUADRATURE BASED REGULARIZATION METHOD FOR ILL-POSED INTEGRAL EQUATIONS

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ABSTRACT. We discuss a particular regularized approximation method for getting stable approximate solutions for the ill-posed integral equations of the form

$$\int_\Omega k(s,t)x(t)\,dt = y(s), \quad x \in L^2(\Omega), \quad s \in \Omega, \qquad (*)$$

where $\Omega := [a,b]$, $k(\cdot,\cdot) \in C(\Omega \times \Omega)$ and $y \in L^2(\Omega)$. The method is based on a convergent quadrature rule, called *regularized Nyström Method* for the Tikhonov regularization of the above ill-posed integral equation, namely,

$$(K^*K + \alpha I)\tilde{x}_\alpha = K^*y,$$

for $\alpha > 0$, where K is the integral operator involved in the equation $(*)$. The idea is that, using the continuity of the kernel function $k(\cdot,\cdot)$, the above regularized equation can be viewed in the setting of $C(\Omega)$ and the integral operator K^*K can be approximated using its Nyström counter part using a convergent quadrature rule to obtain finite rank approximations. It is seen that, for large enough level of quadrature approximation, the derived error estimate is of the same order-optimal accuracy known for the Tikhonov regularization.

REFERENCES

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