Ph. D. Thesis Defence

Energy Stable and Asymptotic Preserving Schemes for the Singular Limits of Some Hyperbolic Systems

By

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Hydrodynamic models of fluids and plasmas seen in nature or laboratory configurations often exhibit phenomena that involve multiple scales in time and space. For the Euler and Euler-Poisson systems that govern the inviscid, compressible flows of fluids and plasma fluids, the three major non-dimensional parameters which quantify the multiscale nature and the singular behaviour of solutions are the Mach number, Froude number and scaled Debye length. Singularly perturbed hydrodynamic models and their singular limits arising from scaling parameters are objects of interest in the mathematical literature. Their numerical approximation poses several challenges and classical explicit time-stepping methods for hyperbolic conservation laws turn out to be inadequate due to the stiffness and the inability to capture a consistent numerical approximation of the limit. A cure to this ailment is the so-called 'Asymptotic Preserving' (AP) methodology. AP schemes are capable of efficiently resolving the different asymptotic regimes of a multiscale problem and reduce the computational cost at the same time. In this thesis, we design and analyse semi-implicit, energy stable, AP and structure preserving finite volume schemes for three singular limits of the scaled, barotropic Euler system with and without source terms, namely the incompressible limit, quasineutral limit and anelastic limit. Detailed analyses are carried out to prove several desirable features of the schemes, such as the consistency with weak solutions, the positivity of density and well-balancing when the underlying system admits steady states. Energy stability is the key feature of the schemes presented and the resulting apriori estimates are used to prove the different asymptotic limits. The results of extensive numerical case studies are presented to corroborate the theoretical claims.

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