The size of the Schur multiplier of special p-groups and other classes of p-groups using graph theory

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The thesis consists of three parts:

I An upper bound on the number of triangles in a graph with fixed number of edges.

II An upper bound on the size of the Schur multiplier of special p-groups of arbitrary rank.

III An upper bound on the size of the Schur multiplier of p-groups of nilpotency class greater than or equal to three.

One of the endeavours in the study of Schur multipliers is to obtain the bounds on the order of the Schur multiplier of different types of groups. The Schur multiplier of a group G is the second homology group $H_2(G, \mathbb{Z})$ of G with integer coefficients. In the first part of the thesis, we solve a problem in extremal graph theory. We obtain bounds on the number of triangles in a graph with fixed number of edges. In the second part of the thesis, we use this graph theory result in the study of p-groups by obtaining bounds on the size of the Schur multiplier of special p-groups of arbitrary rank k. In the third part of the thesis, we obtain bounds on the size of the Schur multiplier of p-groups with nilpotency class greater than or equal to three. We also derive bounds for the order of the Schur multiplier of p-groups based on the coclass of the group.

References:

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2. Sathasivam Kalithasan, Tony Nixon Mavely, and Viji Z Thomas, *On the size of the Schur multiplier of finite groups*, arXiv e-prints (2024), arXiv:2309.02793