ABSTRACT

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The class group and class number of a number field is a mysterious object which helps to measure how far the ring of integers of the number field is from being a unique factorization domain. Hence, many are interested in studying the divisibility and indivisibility properties of the class numbers. Iizuka conjectured that, for any natural number n and prime p, we can find infinitely many d such that the class numbers of the quadratic number fields $\mathbb{Q}(\sqrt{d}), \mathbb{Q}(\sqrt{d+1}), \ldots, \mathbb{Q}(\sqrt{d+n})$ are simultaneously divisible by p. For each odd number k, we provide infinitely many d such that the class numbers of $\mathbb{Q}(\sqrt{d})$ and $\mathbb{Q}(\sqrt{d+1})$ are simultaneously divisible by k by proving certain class number divisibility properties of the number fields in the form of $\mathbb{Q}(\sqrt{1-2m^k})$.

It is a natural question to ask about the simultaneous indivisibility of the class number of the number fields by any k. In this direction, we produced infinitely many d such that the class numbers of the quadratic number fields $\mathbb{Q}(\sqrt{d}), \mathbb{Q}(\sqrt{d+1}), \ldots, \mathbb{Q}(\sqrt{d+n})$ are not divisible by 3^k, where $n = 3^{k+1} - 5$.

Using the connection between the class number of a biquadratic field with the class numbers of it's quadratic sub fields, we prove certain divisibility and indivisibility results for the class numbers of some family of imaginary biquadratic field.

J.P. Serre showed that for any integer m, $a(n) \equiv 0 \pmod{m}$ for almost all n, where $a(n)$ is the n^{th} Fourier coefficient of any modular form with rational coefficients. In this article, we consider a certain class of cuspforms and study $\#\{a(n) \pmod{m}\}_{n \leq x}$ over the set of integers with $O(1)$ many prime factors. Moreover, we show that any residue class $a \in \mathbb{Z}/m\mathbb{Z}$ can be written as the sum of at most thirteen Fourier coefficients, which are polynomially bounded as a function of m.

References

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