"What Does Functional Analysis Have to Do with AI?" From Stone–Weierstrass to the Painting of AI

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Abstract

Have you ever wondered why we study compact spaces, Banach algebras, or the Stone–Weierstrass theorem in Functional Analysis? What do abstract notions of approximation and function algebras have to do with solving practical, real-world challenges? And why can neural networks learn *all kinds of training datasets*, no matter how complex they seem?
In this talk, we will discover how classical results like the Weierstrass and Stone–Weierstrass theorems provide the theoretical foundation for the *Universal Approximation Theorem* — the reason fully connected neural networks are so powerful. What role do compactness, separability, and algebraic structure play in deep learning theory? And how does this pure mathematics guide our understanding of neural network expressivity and its limits?
Finally, what if all this beautiful mathematics could explain not just theory, but creativity itself? We will see how these ideas come alive in *Generative Adversarial Networks (GANs)* — the architectures that power breathtaking AI-generated art, including Studio Ghibli–style images created by ChatGPT. So the journey from compact spaces and function algebras ends with something magical: mathematics revealing how machines learn to paint.

Disclaimer. Don't worry if Functional Analysis feels intimidating: about two-thirds of this talk requires no prerequisites and can be followed by anyone curious about AI. Only the one-third assumes some familiarity with Functional Analysis.

References

[1] Midhun T. Augustine, *A Survey on Universal Approximation Theorems*, arXiv:2407.12895, 2024.

[2] Halsey Royden and Patrick Fitzpatrick, *Real Analysis*, 4th Edition, Pearson, 2010.

[3] Ian Goodfellow, Yoshua Bengio, and Aaron Courville, *Deep Learning*, MIT Press, 2016. deeplearningbook.org

[4] Ian J. Goodfellow et al., *Generative Adversarial Networks*, arXiv:1406.2661, 2014.

[5] Bing Li *et al.*, *AniGAN: Style-Guided Generative Adversarial Networks for Unsupervised Anime Face Generation*, arXiv:2102.12593, 2021.

[6] Yuxin Jiang *et al.*, *Balanced Image Stylization with Style Matching Score*, arXiv:2503.07601, 2025.

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